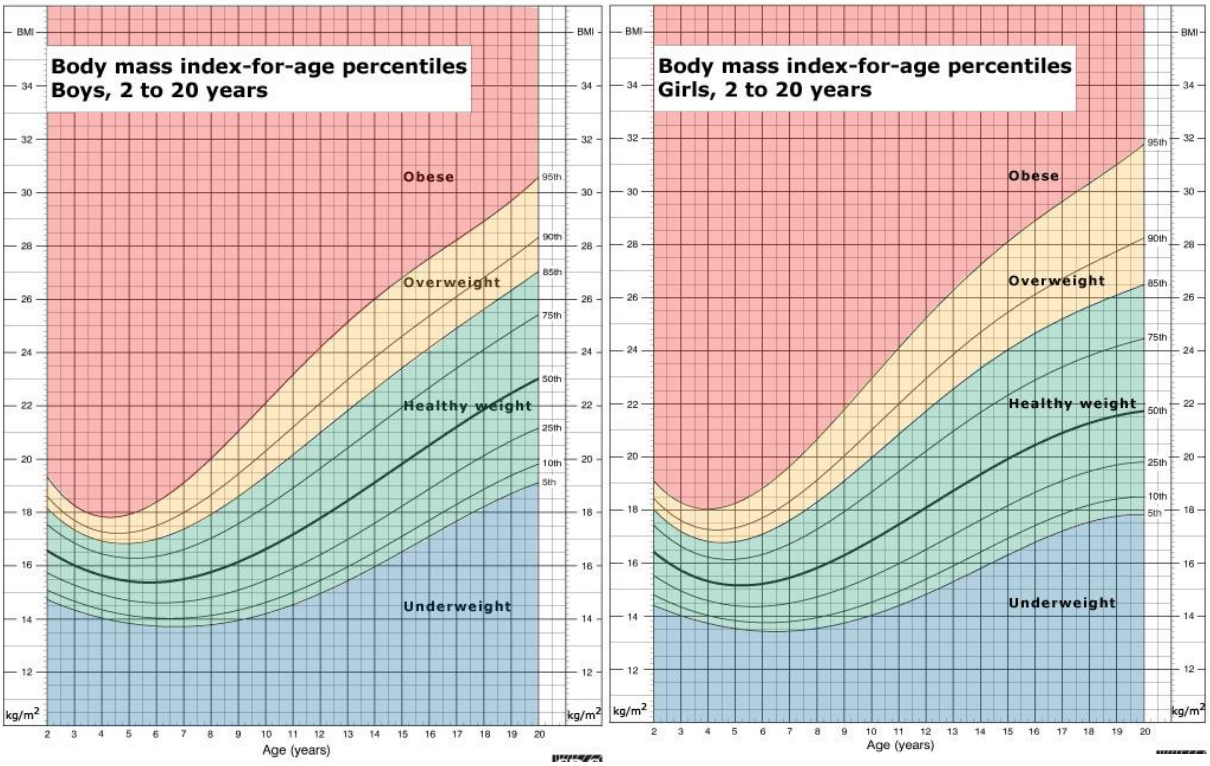
2. Powers and roots

BODY MASS INDEX FOR CHILDREN AND ADOLESCENT

The BMI is used to measure whether a person is underweight, normal weight, overweight or obese.

The formula to calculate the BMI is: $\boxed{\text{Body Mass Index} = BMI = \dfrac{weight\ (kg)}{height^2\ (m^2)}}$

Your situation is determined by the following charts:



What is the BMI for a 15-year-old female who weighs 45 kg and is 1'60m tall?
What is her weight according to the chart?

A girl 1'60 m tall and 13 years old, what would be her weight in kg to have a healthy BMI?

A WHEAT SEED

A wheat seed sown in the soil produces 80 new seeds in a year. If we use all the collected seeds to get as much wheat as possible, how many grains will we have 5 years later from each grain of wheat planted?

SOURCES FOR BMI

You can see this chart on the website:
http://www.betterhealth.vic.gov.au/bhcv2/bhcsite.nsf/pages/bmi4child

You can complete the information about the BMI on the website:
http://www.betterhealth.vic.gov.au/bhcv2/bhcarticles.nsf/pages/Body_Mass_Index_(BMI)

1. POWERS

A power is the product of multiplying a number many times by itself. $a^n = b$

a is the base.

n is the exponent.

b is the power. That is the result.

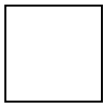
For 3^2; we can say:

Three to the power of 2.
Three squared.
Three to the second power.

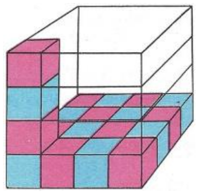
Three raised to the power 2.
Three raised to the second power.
Three to the second.

For 5^3; we can say also five cubed.

The expression $b^2 = b \cdot b$ is called the <u>square</u> of b because the area of a square with side-length b is b^2. It is pronounced "b squared".



The expression $b^3 = b \cdot b \cdot b$ is called the <u>cube</u> of b because the volume of a cube with side-length b is b^3. It is pronounced "b cubed".



The exponent says how many times the base is multiplied by itself.

For example, $4^3 = 4 \cdot 4 \cdot 4 = 64$.

The base 4 appears 3 times in the repeated multiplication, because the exponent is 3.

Here, 4 is the *base*, 3 is the *exponent*, and 64 is the *power*.

3^5 is typically pronounced "three to the fifth" or "three to the five".

PERFECT SQUARES

They are the square of the natural numbers. Complete the list to 20^2.

0; 1; 4; 9; 16;

PERFECT CUBES

They are the cubes of the natural numbers. Complete the list to 10^3.

0; 1; 8; 27; 64;

2. POWERS OF TEN

The powers of ten are the most important powers.

Eg: 10^3=1000; 10^5=100000

We check that the number of noughts is equal to the exponent.

POWERS OF TEN

This is a way to write very big numbers.

It has two parts: the significant figures and the power of ten.

E.g.: The distance between the Earth and the Sun is 152000000 km.

We can write in a scientific notation $152 \cdot 10^6$ km.

And in the opposite way: A light year is $9 \cdot 10^{12}$ km = 9 000 000 000 000 km.

This is the distance that a beam of light travels in a year.

3. PROPERTIES OF POWERS

PRODUCT RULE. PRODUCT OF POWERS OF THE SAME BASE

To multiply powers of the same base you add their exponents.

$$a^n \cdot a^m = a^{n+m}$$

Example:

$10^3 \cdot 10^5 =$

QUOTIENT RULE. QUOTIENT OF POWERS OF THE SAME BASE

To divide powers of the same base we subtract the exponents.
(Subtract the bottom exponent from the top exponent)

$$a^n : a^m = a^{n-m}$$

Example:

$10^7 : 10^5 =$

Consequences

EXPONENT 0

Any number to the power of zero is one.

$$a^0 = 1$$

$5^0 = 1$. Because, $5^0 = 5^2 : 5^2 = 1$.

POWER OF A POWER.

When you raise a power to an exponent you multiply those exponents together.

$$\left(a^n\right)^m = a^{n.m}$$

Example:

$\left(10^2\right)^3 =$

POWER OF A PRODUCT

When you have a product raised to an exponent raise each factor to that exponent.

$(a \cdot b)^n = a^n \cdot b^n$ and in the opposite way: $a^n \cdot b^n = (a \cdot b)^n$

For example, $5^4 \cdot 2^4 = (5 \cdot 2)^4 = 10^4 = 10000$

POWER OF A QUOTIENT

When you have a quotient raised to an exponent raise each number to that exponent.

$$(a:b)^n = a^n : b^n \text{ and in the opposite way: } a^n : b^n = (a:b)^n$$

For example, $15^3 : 5^3 = (15:5)^3 = 3^3 = 27$

4. SQUARE ROOT

If 'b' is the square root of 'a' then 'b' squared is 'a'.

$$\sqrt[2]{a} = b \;\; means\ that\ b^2 = a$$

'2' is the index.
'a' is the radicand.
'b' is the radical.
In a square, we have the following relations:

Area = side^2

Area is the side squared.
In the opposite way:

$$Side = \sqrt{area}$$

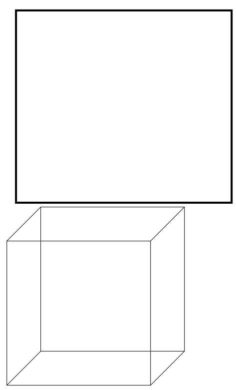
The side is the square root of the area.

CUBIC ROOTS

If the cube root of 'a' is 'b' it means that 'b' cubed is 'a'.

$$\sqrt[3]{a} = b \;\; means\ that\ b^3 = a$$

'3' is the index.
'a' is the radicand.
'b' is the radical.
In a cube we have the following relations:



Volume = edge^3

Volume is the edge cubed.
In the opposite way:

$$Edge = \sqrt{volume}$$

The edge is the cube root of the volume.

WHOLE ROOTS

They are the roots with a whole result. That is, the radicals whose radicand is a perfect square.

For instance, $\sqrt{81} = 9$. The square root of 81 is a whole root because the result is an integer.

Example:

Could you write three more whole square roots?

ESTIMATING RADICALS

Per the charts estimate the value of the following radicals:

- $\sqrt{13} \approx$
- $\sqrt{34} \approx$
- $\sqrt{65} \approx$
- $\sqrt{172} \approx$
- $\sqrt{348} \approx$
- $\sqrt[3]{49} \approx$
- $\sqrt[3]{412} \approx$

Cuadrados

n	n^2	n	n^2
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Cubos

n	n^3
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

ORDER OF OPERATIONS. PEDMAS

Parenthesis
Exponents and roots
Division / Multiplication
Addition / Subtraction

THEORY SUMMARY

Power

A power is the product of multiplying a number many times by itself.
$a^n = b$; a is the base; n is the exponent; b is the power; that is the result.

Perfect square

They are the square of the natural numbers.

Perfect cubes

They are the cubes of the natural numbers.

Scientific notation

This is a way to write very big numbers.
It has two parts: the significant figures and the power of ten.

Product of powers of the same base

To multiply powers of the same base you add their exponents.

Quotient of powers of the same base

To divide powers of the same base we subtract the exponents.

Exponent 0

Any number to the power of zero is one.

Power of a power

To raise a power to an exponent you multiply those exponents together.

Square root

If ‘b’ is the square root of ‘a’ then ‘b’ squared is ‘a’.

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Whole roots

They are the roots with a whole result.

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EXERCISES AND PROBLEMS

1. Powers

1. Calculate: $a)\ 3^5;\ \ b)\ 2^6; c)5^0$

2. Powers of ten

2. Write in scientific notation:
a) 378000000; b) 57120000000

3. Write as a power of ten:
a) 1 thousand; b) 100 millions; c) 1 thousandth; e) 1 ten-thousandth

4. Write as a power of ten:
a) 548 millions; b) 235 thousandths; c) 123 millionths; d) 248 hundredths.

3. Properties of powers

5. Simplify the following expressions writing as a unique power using the properties of powers:
$a)3\ \ \cdot 3^6\ \ b)x^7 : x^4\ \ c)\left(a^2\right)^3\ \ d)2^4 \cdot 2^2 \cdot 2^5$

6. Write as a unique power the following expressions:

$$a)\ 10^3 \cdot 10^5; b)\ 10^2 \cdot 10^6; c)\ 10^5 : 10^7; d)\ \left(10^2\right)^3$$

7. Simplify the following expressions writing as a unique power using the properties of powers:
$a)\ 4^2 \cdot 4^5\ \ b)\ x^6 : x\ c)\ \left(a^2\right)^4\ \ d)\ 2^4 \cdot 2^3 \cdot 2^5$

8. Simplify the following expressions using the properties of powers:
$a)3^0 \cdot 3^4 =\ b)\ x^3 : x^2\ \ c)\ \left(x^5\right)^0\ \ d)\ 2^3 \cdot 2^4 \cdot 2$

9. Simplify the following expressions using the properties of powers:
$a)3^4 : 3^0 =\ b)\ x^3 \cdot x^2\ \ c)\ \left(x^5\right)^0\ \ d)\left(2^3 \cdot 2^4 \cdot 2\right) : 2^5$

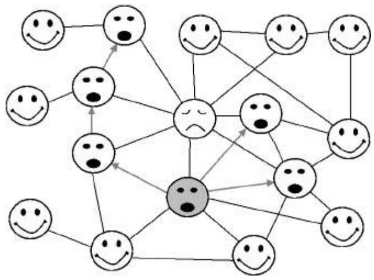
10. Simplify the following expressions: $a)3^{11} : 3^7 =\ b)\ x^5 \cdot x^2 \cdot x\ \ c)\left(x^0\right)^5\ \ d)\left(2^4 \cdot 2^2 \cdot 2\right)^2$

11. Simplify the following expressions using the properties of powers:
$a)5^2 \cdot 5^0 \cdot 5^3 =\ b)\ x^3 : x^3\ \ c)\ \left(x^0\right)^2\ \ d)\ \left(2^3 \cdot 2^4 \cdot 2\right) : \left(2^5 \cdot 2^3\right)$

12. Our heart beats 80 times per minute. How many times does your heart beat per year? Write as a power of ten.

13. A sheet of paper is 1 mm wide. You double the sheet and after that once again and so on. After you do this action twenty times, how many cm is the paper in width? Write as a power of ten.

14. A girl tells some gossip to two friends. The next day the two friends do the same and so on. How many people know the gossip fifteen days later?



15. How many seconds does a century have? Write the result as a power of ten.

3. Square root

16. Find out the value of the following calculations: $\left(2+2^4\cdot 3-5\right)\sqrt{9}+7$

17. Calculate: $4+\sqrt{25-9}\left(2+2^4\cdot 3+5\right)+3$

18. Calculate: $\sqrt{25-16}\cdot\left(7\cdot 5-2\cdot 3^2+5\right)+2$

19. Calculate: $4+\sqrt{16+9}-\left(5+2\cdot 3^2-15:3\right):2$

20. How many metres of fence do you need to surround a squared plot whose area is 900 m^2